Embedded Microstrip

Impedance Formula

Doug Brooks

Every so often questions about the formula for the impedance of embedded microstrip traces come up. Questions arise for several reasons. One is that there isn't much really good information available about this formula. But another reason is that the primary source of information most of us use (the Institute for Interconnecting and Packaging Electronic Circuits, or IPC) is wrong about it!

I am going to start the discussion with the IPC's development of the formula for regular microstrip. Then, you will be able to see the problem that exists with their extension of this derivation to the embedded microstrip case. (Note: All numerical references herein are from IPC-D-317A, "Design Guidelines for Electronic Packaging Utilizing High-Speed Techniques. Other IPC publications (e.g. IPC-D-275 and IPC-2221) have similar sections with similar errors.)

Microstrip Impedance:

In the development of the formula for the impedance of a microstrip trace, IPC, in section 5.5.1.1 starts with the impedance of a single bare wire near ground:

Equation 5.29:

$$Z_0 = \frac{60}{\sqrt{e_r}} Ln\left(\frac{4H}{d}\right)$$

where e_r is the relative permittivity of the medium (dielectric) surrounding the wire, H is the height above the ground, and d is the diameter of the wire.

In order to get to the microstrip equation, two adjustments to this formula are required. First is the adjustment for the fact that the wire is not surrounded by a homogeneous medium. In fact, it has a dielectric below it and air above it. The adjustment for this is to change e_r to e'_r where:

Eq. 5.30

$$e'_r = 0.475 e_r + 0.67$$

The *second* adjustment accounts for the fact that a trace is flat and rectangular, not round. This adjustment substitutes

Equation DGB1
5.98H/(.8W+T) for (4H/d)

in Equation 5.29, where W and T are the width and thickness, respectively, of the rectangular trace. These two adjustments lead to:

Equation DGB2:

$$Z_0 = \frac{60}{\sqrt{.475 e_r + .67}} Ln\left(\frac{5.98H}{(.8W+T)}\right)$$

This equation is the same as, and reduces to, IPC's Equation 5.32 and the equation shown in 5.34 (Hint: divide numerator and denominator by the square root of .475):

Equation 5.32 and 5.34

$$Z_0 = \frac{87}{\sqrt{e_r + 1.41}} Ln\left[\frac{5.98H}{(.8W+T)}\right]$$

Equation 5.32 is the equation we usually rely on for the impedance of a microstrip trace.

Embedded Microstrip:

In the next section, the IPC standard says: "The equations for embedded microstrip *are the same* as in the section on (uncoated) microstrip, with a modified effective permittivity." (emphasis added) It goes on to say that the adjustment to be made is to substitute e_r' for e_r in Equations 5.32 and 5.34, where:

Equation DGB3:

$$e_r' = e_r\left(1 - e^{\left(\frac{-1.55H_1}{H}\right)}\right)$$

Here the new variable, H1, is the height of the "embedding" above the plane. H1 is greater than H, and the difference between them, H1-H, is approximately the thickness of the "embedding."

But note what happens at the extremes of H1. When H1 is very large (deeply embedded), Equation DGB3 reduces simply to e_r, which results in a value for Equation 5.34 that is correct for *no embedding*, not deep embedding! And if

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H1 is nearly H (i.e. no embedding), then DGB3 reduces to a calculated value that is somewhat similar to what would be found from Equation 5.30 (depending on the combination of other values), and the values calculated from Equation 5.34 change dramatically. Thus, this IPC adjustment (substituting DGB3 into Equation 5.34) leads to exactly the *opposite* result from what would be intuitive.

The *correct* statement --- what the standard *should* say --- is, “The ***development*** of the equations for embedded microstrip are the same as in the section on (uncoated) microstrip…” We should not substitute Equation DGB3 as an adjustment into the equation for the impedance of a microstrip trace (Equation 5.32 or 5.34), which has *already* been adjusted for e_r, but rather as a separate, *alternative* adjustment into the *beginning* of the derivation, Equation 5.29. Thus, when that equation (Equation 5.29) is adjusted for (1) embedding and (2) rectangular instead of round wire, the correct equation for embedded microstrip becomes Equation DGB4:

Equation DGB4:

$$Z_0 = \frac{60}{\sqrt{e_r\left(1 - e^{\left(\frac{-1.55H_1}{H}\right)}\right)}} Ln\left[\frac{5.98H}{.8W + T}\right]$$

Discussion:

It is interesting now to substitute values for H1 into Equation DGB4 that reflect either no embedding or very large embedding. Results are obtained that are roughly equivalent to those that would be calculated from the pure microstrip equation or from a stripline equation with one plane very far away. This is much more intuitively satisfying than the result given in the IPC publications. The formulas do not agree exactly because (1) their derivations are different and (2) we are projecting the embedded microstrip equation beyond its range of applicability.

The formula for embedded microstrip is an approximation, at best. The original work was done on this a long time ago under test conditions apparently long forgotten. The results are *highly* sensitive to the values of the variables. In particular, it assumes all the dielectric is uniform throughout, but in reality the coating above the trace very often has a different e_r than the material under the trace. Therefore, as with all approximating equations, we are well advised to use them as guidelines and always talk to whoever the board fabricator will be if more accurate results are necessary.