

Differential Impedance

What's the Difference?

Douglas Brooks

Just when you thought you had mastered Z_0 , the characteristic impedance of a PCB trace, along comes a data sheet that tells you to design for a specific differential impedance. And to make things tougher, it says things like: "... since the coupling of two traces can lower the effective impedance, use 50 Ohm design rules to achieve a differential impedance of approximately 80 Ohms!" Is that confusing or what!!

This article shows you what differential impedance is. But more than that, it discusses **why** it is, and shows you how to make the correct calculations.

Single Trace:

Figure 1(a) illustrates a typical, individual trace. It has a characteristic impedance, Z_0 , and carries a current, i . The voltage along it, at any point, is (from Ohm's law) $V = Z_0 * i$.

General case, trace pair:

Figure 1(b) illustrates a pair of traces. Trace 1 has a characteristic impedance Z_{11} , which corresponds to Z_0 , above, and current i_1 . Trace 2 is similarly defined. As we bring Trace 2 closer to Trace 1, current from Trace 2 begins to couple into Trace 1 with a proportionality constant, k . Similarly, Trace 1's current, i_1 , begins to couple into Trace 2 with the same proportionality constant. The voltage on each trace, at any point, again from Ohm's law, is:

$$V_1 = Z_{11} * i_1 + Z_{11} * k * i_2 \quad \text{Eqs. 1}$$

$$V_2 = Z_{22} * i_2 + Z_{22} * k * i_1$$

Now let's define $Z_{12} = k * Z_{11}$ and $Z_{21} = k * Z_{22}$. Then, Eqs. 1 can be written as:

$$V_1 = Z_{11} * i_1 + Z_{12} * i_2 \quad \text{Eqs. 2}$$

$$V_2 = Z_{21} * i_1 + Z_{22} * i_2$$

This is the familiar pair of simultaneous equations we often see in texts. The equations can be generalized into an arbitrary number of traces, and they can be expressed in a matrix form that is familiar to many of you.

Special case, differential pair:

Figure 1(c) illustrates a differential pair of traces. Repeating Equations 1:

$$V_1 = Z_{11} * i_1 + Z_{11} * k * i_2 \quad \text{Eqs. 1}$$

$$V_2 = Z_{22} * i_2 + Z_{22} * k * i_1$$

Now, note that in a carefully designed and balanced situation,

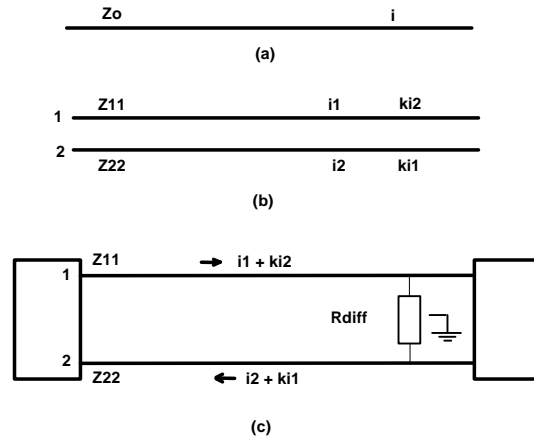


Figure 1
Various Trace Configurations

$$Z_{11} = Z_{22} = Z_0, \text{ and}$$

$$i_2 = -i_1$$

This leads (with a little manipulation) to:

$$V_1 = Z_0 * i_1 * (1-k) \quad \text{Eqs. 3}$$

$$V_2 = -Z_0 * i_1 * (1-k)$$

Note that $V_1 = -V_2$, which we already knew, of course, since this is a differential pair.

Effective (odd mode) impedance:

The voltage, V_1 , is referenced with respect to ground. The effective impedance of Trace 1 (when taken alone this is called the "odd mode" impedance in the case of differential pairs, or "single mode" impedance in general) is voltage divided by current, or:

$$Z_{\text{odd}} = V_1 / i_1 = Z_0 * (1-k)$$

And since (from above) $Z_0 = Z_{11}$ and $k = Z_{12} / Z_{11}$, this can be rewritten as:

$$Z_{\text{odd}} = Z_{11} - Z_{12}$$

which is a form also seen in many textbooks.

The proper termination of this trace, to prevent reflections, is with a resistor whose value is Z_{odd} . Similarly, the odd mode impedance of Trace 2 turns out to be the same (in this special case of a balanced differential pair).

Differential impedance:

Assume for a moment that we have terminated both traces in a resistor to ground. Since $i_1 = -i_2$, there would be no current **at all** through ground. Therefore, there is no real reason to connect the resistors to ground. In fact, some people would argue that you must not connect them to ground in order to isolate the differential signal pair from ground noise. So the normal connection would be as shown in Figure 1(c), a single resistor from Trace 1 to Trace 2. The value of this resistor would be the sum of the odd mode impedance for Trace 1 and Trace 2, or

$$Z_{diff} = 2 * Z_o * (1-k) \text{ or} \\ 2 * (Z_{11} - Z_{12})$$

This is why you often see references to the fact that a differential pair of traces can have a differential impedance of around 80 Ohms when each trace, individually, is a 50 Ohm trace.

Calculations:

To say that Z_{diff} is $2*(Z_{11} - Z_{12})$ isn't very helpful when the value of Z_{12} is unintuitive. But when we see that Z_{12} is related to k , the coupling coefficient, things can become more clear. In fact, this coupling coefficient is the same coupling coefficient I talked about in my Brookspeak column on crosstalk (Footnote 1). National Semiconductor has published formulas for Z_{diff} that have become accepted by many (Footnote 2):

$$Z_{diff} = 2*Z_o[1-.48*\exp(-.96*S/H)] \text{ (Microstrip)}$$

$$Z_{diff} = 2*Z_o[1-.347*\exp(-2.9*S/H)] \text{ (Stripline)}$$

where the terms are as defined in **Figure 2** and "exp()" means e , the base of the natural logarithm, raised to the power in the parentheses. Z_o is as traditionally defined (Footnote 3).

Common Mode Impedance:

Just to round out the discussion, common mode impedance differs only slightly from the above. The first difference is that $i_1 = i_2$ (without the minus sign.) Thus Eqs. 3 become

$$V_1 = Z_o * i_1 * (1+k) \text{ Eqs. 4}$$

$$V_2 = Z_o * i_1 * (1+k)$$

and $V_1 = V_2$, as expected. The individual trace impedance, therefore, is $Z_o*(1+k)$. In a common mode case, both trace terminating resistors **are** connected to ground, so the current through ground is i_1+i_2 and the two resistors appear (to the device) in parallel. Therefore, the common mode impedance is the parallel combination of these resistors, or

$$Z_{common} = (1/2)*Z_o*(1+k), \text{ or}$$

$$Z_{common} = (1/2)*(Z_{11} + Z_{12})$$

Note, therefore, that the common mode impedance is approximately $1/4$ the differential mode impedance for trace pairs.

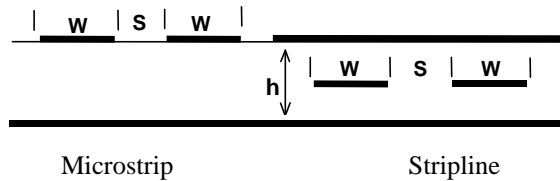


Figure 2
Definition of terms for Differential impedance calculations

Footnotes:

1. "Crosstalk, Part 2: How Loud Is It?" Brookspeak, December, 1997.
2. See National Semiconductor's "Introduction to LVDS" (page 28-29) available from their web site at www.national.com/appinfo/lvds/ or their "Transmission Line RAPIDESIGNER Operation and Applications Guide", Application Note 905.
1. See "PCB Impedance Control, Formulas and Resources", Printed Circuit Design, March, 1998, p12. The formulas are:

$$Z_o = 87 * \text{Ln}[5.98H / (.8W + T)] / \text{SQR}(\epsilon_r + 1.41) \text{ (Microstrip)}$$

$$Z_o = 60 * \text{Ln}[1.9(H) / (.8W + T)] / \text{SQR}(\epsilon_r) \text{ (Stripline)}$$